Appendix I

We wish to relate the coefficients B, B_1, in Eq. (IV-4) to A and A_1 in Eq. (IV-1). For small warping we may express the energy in the form

$$E = \frac{\hbar^2 k_o^2}{m^*} \left[\frac{1}{2}(k/k_o)^2 + r\,(k/k_o)^4\, Y_4(\theta,\phi) + s\,(k/k_o)^6\, Y_6(\theta,\phi)\right] \qquad \text{(A-1)}$$

which is just Eq. (I-4) with $s = rt$.

Furthermore

$$\left(\frac{\partial E}{\partial k}\right)_{E_F} = \frac{\hbar^2 k_o}{m^*}\left[\left(\frac{k}{k_o}\right) + 4r\left(\frac{k}{k_o}\right)^3 Y_4(\theta,\phi) + 6s\left(\frac{k}{k_o}\right)^5 Y_6(\theta,\phi)\right] \qquad \text{(A-2)}$$

where the derivative is evaluated at the Fermi energy.

Now we know that

$$\left(\frac{\partial E}{\partial k}\right)_{E_F}\left(\frac{\partial k}{\partial E}\right)_{E_F} = 1 \qquad \text{(A-3)}$$

in any direction in k space. Equation (IV-4) gives $\left(\frac{\partial k}{\partial E}\right)_{E_F}$.

We use the subscripts 1, 2, 3 to indicate the [100], [110], and [111] directions: the Kubic harmonics and $\left(\frac{k}{k_o}\right)$ are evaluated in these directions. For example

$$k_1 = \left(\frac{k}{k_o}\right)_1 = (1 + A + A_1) \qquad \text{(A-4)}$$

using $Y_4[100] = Y_6[100] = 1$ in Eq. (IV-1).

Let us introduce the notation

$$\left[\left(\frac{k}{k_o}\right)_i + 4r\left(\frac{k}{k_o}\right)_i^3 Y_4(i) + 6s\left(\frac{k}{k_o}\right)_i^5 Y_6(i)\right] = a_i\,(A, A_1, r, s) \qquad \text{(A-5)}$$

where i runs from 1 to 3.

Substituting Eqs. (A-2), (A-5) and (IV-2) into Eq. (A-3) we obtain

$$(1 + B + B_1)\,a_1 = \left(1 - \frac{B}{4} - \frac{13}{8}B_1\right) a_2 = \left(1 - \frac{2B}{3} + \frac{16}{9}B_1\right) a_3 \qquad \text{(A-6)}$$

From this we obtain two linear equations for B and B_1 whose solution is

$$B = \frac{\begin{vmatrix} (\alpha_2 - \alpha_1) & (\alpha_1 + \frac{13\alpha_2}{8}) \\ (\alpha_3 - \alpha_1) & (\alpha_1 - \frac{16\alpha_3}{9}) \end{vmatrix}}{\begin{vmatrix} (\alpha_1 + \frac{\alpha_2}{4}) & (\alpha_1 + \frac{13\alpha_2}{8}) \\ (\alpha_1 + \frac{2\alpha_3}{3}) & (\alpha_1 - \frac{16\alpha_3}{9}) \end{vmatrix}}$$

$$B_1 = \frac{\begin{vmatrix} (\alpha_1 + \frac{\alpha_2}{4}) & (\alpha_2 - \alpha_1) \\ (\alpha_1 + \frac{2\alpha_3}{3}) & (\alpha_3 - \alpha_1) \end{vmatrix}}{\begin{vmatrix} (\alpha_1 + \frac{\alpha_2}{4}) & (\alpha_1 + \frac{13\alpha_2}{8}) \\ (\alpha_1 + \frac{2\alpha_3}{3}) & (\alpha_1 - \frac{16\alpha_3}{9}) \end{vmatrix}}$$

The α_i depend on r and s of Eq. (A-1); we now obtain these. We substitute

$$(\frac{k}{k_o}) = 1 + A\ Y_4\ (\theta,\phi) + A_1\ Y_6\ (\theta,\phi) = k_i \qquad \text{(IV-1)}$$

into the expression for the energy, Eq. (A-1). The energy must be constant on the Fermi surface. By requiring the energy in the three principal directions to be the same, we obtain

$$1/2\, k_1^{\ 2} + r\, k_1^{\ 4} + s k_1^{\ 6} = 1/2\, k_2^{\ 2} - 1/4\, r\, k_2^{\ 4} - \frac{13}{8}\ s k_2^{\ 6} =$$

$$1/2\, k_3^{\ 2} - 2/3\ r\, k_3^{\ 4} + \frac{16}{9}\ s k_3^{\ 6}\,. \qquad \text{(A-7)}$$